# Adaptive Interference Tolerant Receivers for Asynchronous Cooperative MIMO Communications

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Abstract-Adaptive single user receivers are demonstrated in this paper for a Cooperative Virtual MIMO network applying Spread Spectrum Sequences. In asynchronous decentralized cooperative systems, it is expected that, for typical wireless environments, user transmissions from adjacent relaying nodes (and other cells) will create interference. Large scale MIMO antenna arrays can mitigate interference with sufficient degrees of freedom but they can be underdetermined in decentralized non orthogonal multiple access (NOMA). In this paper we use Spread Spectrum CDMA sequences when user cooperation and relaying is necessitated and correspondingly apply decentralized single user algorithms utilizing an adaptive multiuser detection approach. The approach does not require Channel State Information (CSI) and operates in a decentralized manner without requiring knowledge of the transmissions from other users (i.e. power, channel gain and phase, and multiuser chips). This yields a low interference cooperative MIMO communication network that is useful for communication in areas with poor coverage or to temporarily increase spectral efficiency to enable a high throughput uplink or downlink channel. The adaptive algorithm utilized in this paper is investigated for both chip-level and symbol-level optimization where it noted that when applying chip-level optimization, a more interference robust receiver can be built when utilizing processing gain (rather than receiver dimensionality) as a metric to combat interference when the number of transmitter antennas used are fixed.

# *Index Terms*—Adaptive Multiuser Detection, Cooperative MIMO, Relaying.

#### I. INTRODUCTION

**E** MERGING mm-wave technology enables large scale MIMO systems [1] be built into pocket sized devices. They potentially offer enormous spatial computation power including the ability to concurrently multiplex a large number of channels in the same temporal and frequency space while also being capable of suppressing a number of interfering transmissions arising from other terminals. High spatial dimensionality is advantageous to assisting a wireless network combat interference while offering diversity and multiplexing gains. However in Nonorthogonal Decentralized Multiple Access, for instance such as in a frequency division system like OFDMA (where multiple transmissions could reuse some of the frequency space occupied by another channel), an interference floor develops that could overwhelm even a large array.

Charan Litchfield is with West Virginia University Institute of Technology, Department of Electrical and Computer Engineering, Beckley, WV, USA (email: charan.litchfield@mail.wvu.edu) and Triantafyllos Kanakis is with the University of Northampton, Department of Computing, Northhampton, UK (email: Triantafyllos.Kanakis@northampton.ac.uk). While multiple access interference could be circumvented in a centralized cooperative system (i.e. orthogonal transmission schemes), in a decentralized system we cannot expect all user transmissions to be orthogonal, particularly if arising from adjacent cells or user clusters. Interference management is one aspect of the challenge involved in practical wireless cooperative system design. There exist numerous challenges ranging from physical obstructions to line of site, multiple shadowing zones, and issues of interference due to non orthogonal multiple access (NOMA). Cooperative Virtual multiple antenna systems [2] [3] are an extension to the relaying theme - able to assist the wireless network combat some aspects of path loss, shadowing and outages. A Virtual Cooperative MIMO system can increase the spatial diversity and multiplexing gain [4] [5], but also offer significant capacity when shadowing (where antenna's on a small device instantly fade) occurs and roaming causes a user terminal near the cell edge to be more susceptible to interference and the near far effect. Naturally, implementing a cooperative network, particularly one that is decentralized, yields a potential interference problem arising from other nodes and adjacent cells. The work in this paper follows up the research we conducted in [7] [8] to broaden the topic to SHF Spread Spectrum CDMA environments and consider adaptive receivers [11] that, with beamforming assistance, can mitigate interference from other decentralized terminals. We consider interference in a cellular environment with frequency reuse of one. In context of an asynchronous and decentralized MIMO cooperative relay with unknown and uncontrollable sources of interference, receivers built with adaptive multiuser detection filters are investigated for two cases - namely where chip matched filtering is applied prior to array optimization (which yields a computationally efficient receiver but one which is bound by the array dimensionality), and when joint array and multiuser detection filter optimization is applied (yielding higher complexity but more degrees of freedom to mitigate interference). For the second receiver, if the multiplexing gain of the transmitting array is fixed, interference tolerance scales with both receiver antenna dimensionality and processing gain, while the former is more reliant on array dimensionality. The paper is organized as follows: In section II, the System Model is developed. Section III offers the analysis and development of the algorithm employed in this paper. Sections IV and V yield the Simulation Model and Results, and section VI Concludes this study.

## II. SYSTEM MODEL

For the Cooperative MIMO channel we will form a baseband equivalent time invariant model. We assume Nyquist pulse shapes P(t) are sent and overlapped by a time variable,  $\tau$  with single sided Bandwidth  $W = \frac{1}{\tau}$ . We assume a complex symbol alphabet is transmitted, the baseband signal at the  $n^{th}$ transmit antenna is

$$x_n(t) = \sum_{m=0}^{\infty} \theta_n[m] P(t - m\tau)$$
(1)

with  $\theta_n[m]$  the  $m^{th}$  transmitted single/multichannel chips on the  $n^{th}$  transmit antenna.  $\boldsymbol{\theta}[n] = \mathbf{S}\boldsymbol{\phi}[n]$  which is a scaled combination (via the chip sequence matrix S) of the modulation symbols over the layers.  $\mathbf{S} = [\mathbf{s}_0^T, \mathbf{s}_1^T, \cdots, \mathbf{s}_{K-1}^T] \in \mathbb{C}^{G \times K}$  is the matrix of chips with  $\mathbf{s}_k \in \mathbb{C}^{1 \times G}$  the chips applied for MIMO layer k and G the number of chips multiplexed per symbol. We denote N as being the number of Transmit Antennas and K the number of spatially multiplexed single user channels. A beamforming matrix transform will be denoted as  $\mathbf{D} \in \mathbb{C}^{N \times K}$ . The number of receiver antenna is denoted as the variable U, where we assume  $U \ge K$ . We can equate  $\theta_n[m] = \boldsymbol{\delta}_n^T \mathbf{D} \boldsymbol{\phi}[m]$  with  $\boldsymbol{\delta}_n$  the Kronecker Delta Vector with a 1 in the  $n^{th}$  position and 0's elsewhere while  $\boldsymbol{\phi} \in \mathbb{C}^{K}$  is denoted by  $\boldsymbol{\phi}[m] = [\theta_{0}[m], \theta_{1}[m], \cdots, \theta_{K-1}[m]]^{T}$ with  $\mathbb C$  the set of complex numbers. To account for transmitter power control (i.e. waterfilling), if we let  $\mathbf{b}[m] \in \Psi^K$ be the transmitted spatially multiplexed modulation symbols (with  $\Psi$  representing the modulation symbol set(s)), then  $\boldsymbol{\phi}[m] = \boldsymbol{\Sigma} \mathbf{b}[m]$  with  $\boldsymbol{\Sigma} = diag[\alpha_0, \alpha_1, ..., \alpha_{K-1}] \in \mathbb{R}^{K \times K}$  a diagonal matrix of real scalars subject to the power constraint  $tr(\boldsymbol{\Sigma}^T \boldsymbol{\Sigma}) \leq P_T$  with  $P_T$  the available power at transmitter and  $tr(\bullet)$  the trace of the matrix. The baseband signal at the  $u^{th}$  receiver antenna after matched filtering is given by

$$r_u(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} h_{n,u} \theta_n[m] P(t-m\tau) * P(-t) + v_u(t)$$
(2)

with  $v_u(t)$  the filtered white noise after receiver matched filter. We therefore note the following baseband Linear Algebraic System Model

$$\mathbf{r}[m] = \mathbf{H} \mathbf{D} \boldsymbol{\theta}[m] + \mathbf{v}[m] \tag{3}$$

In this paper we will further assume  $\mathbf{D} = \mathbf{I}$  with  $\mathbf{I}$  the Identity Matrix and thus N = K. The transmitted chips  $\theta_k[n]$  with  $E[|\theta_k[n]|^2] = \sigma_k^2$  are perturbed by the channel containing two uncertainties, namely the additive white Gaussian noise (AWGN)  $\boldsymbol{\nu} \in \mathbb{C}^U$  (with two sided PSD  $\frac{N_0}{2} = \sigma_{\nu}^2$  and covariance  $\sigma_{\nu}^2 \mathbf{I}$ ), and the multipath propagation channel matrix  $\mathbf{H} \in \mathbb{C}^{U \times N}$  taking the common MIMO form which is shown in (4).

$$\mathbf{H} = \begin{pmatrix} h_{0,0} & h_{1,0} & h_{2,0} \cdots h_{N-1,0} \\ h_{0,1} & h_{1,1} & h_{2,1} \cdots h_{N-1,1} \\ h_{0,2} & h_{1,2} & h_{2,2} \cdots h_{N-1,2} \\ & \ddots & \\ h_{0,U-1} h_{1,U-1} h_{2,U-1} \cdots h_{N-1,U-1} \end{pmatrix}$$
(4)



Fig. 1. General system model of Cooperative MIMO

In Fig.(1) the Cooperative terminals are sorted in Mean Square Error (MSE) by the algorithm in [12], with contention and optimization for relaying resources greedy, and relays transmitting in a SHF/ISM band applying CDMA [9] to assist with interference management. For a Cooperative MIMO Relay channel model, since user terminals are relaying each layer, where we assume number of terminals equals the number of layers K, the model must incorporate two propagation channel components and power correspondingly normalized by the number of hops employed [10]. As such, for the transmission cluster the following model is applied (assumed dimensionality U) for each relay node, with the number of nodes assumed to be K and the number of transmitted antennas employed by the relay also assumed to be K. Hence for the  $k^{th}$  relay node, adopting the channel form in (4), the received signal is  $\mathbf{r}_k[n] = \mathbf{H}_k \mathbf{D}_k \boldsymbol{\theta}[n] + \boldsymbol{\nu}_k[n]$  with  $\mathbf{H}_k$  and  $\mathbf{H}_{k\pm m}$  statistically independent and modelled according to (4). The relay terminal will utilize decode and forward, thus the decoded information symbols **b** will be used to form the chip sequences  $\hat{\theta}[n]$ . To represent the full cluster, the matrices  $\mathbf{H}_k$  are stacked for all K relay terminals into one tall Matrix  $\tilde{\mathbf{H}}$  such that  $\mathbf{r}_k[n] =$  $\tilde{\mathbf{H}}\boldsymbol{\theta}[n] + \mathbf{v}[n]$  with  $\mathbf{v}$  a vector with stacked noise from all the antenna elements  $\nu_{k=0:K-1}$ . We adopt an identical approach regarding the receiver cluster relay to user terminal albeit with recovered information symbols (from the relay) forming the chip sequences, i.e.  $\hat{\theta}[n]$  instead of  $\theta[n]$ . The Relay itself will adopt a similar model where  $\mathbf{H}_r = [\mathbf{H}, \mathbf{H}_I] \in \mathbb{C}^{U \times (K+I)}$  and  $\boldsymbol{\theta} = \begin{bmatrix} \tilde{\boldsymbol{\theta}}^T, \boldsymbol{\theta}_I \end{bmatrix} \in \mathbb{C}^{(K+I) \times 1}$  represent the channel matrix of interest concatenated with the interference channel matrix and the transmitted chips of interest stacked with the interference signal. There are  $N \cdot K$  relay transmitters (including spatial multiplexing) and  $U \cdot K$  total receiver channels, albeit each can only optimize U channels independently. The information capacity is, for decode and forwarding,  $C \leq min [C_s, C_r, C_d]$ with  $C_r$  the information capacity for the Relay Part of the communication channel typically operating as the lower bound (the other Capacity variables being that at the source to relay and relay to destination).

#### **III. ADAPTIVE MULTIUSER DETECTION**

#### A. MMSE Multiuser Detector

The classic MMSE Multiuser Detector [13] is achieved with the conditional mean estimator  $\hat{\mathbf{b}}(r) = E[\mathbf{b}|\mathbf{r}]$  when given a cost function of  $E[|\mathbf{b}-\hat{\mathbf{b}}(\mathbf{r})|^2]$ . Utilizing a linear constraint, the problem is minimizing the functional  $E||\mathbf{b}-\mathbf{Mr}||^2$  with **M** an affine transform that minimizes the mean square error (MSE). If **M** is a  $K \times K$  Matrix  $\mathbf{M} = [\mathbf{m}_0^T, \mathbf{m}_1^T, \cdots, \mathbf{m}_{K-1}^T] \in \mathbb{C}^{K \times K}$ , with the assumption it is full rank, it is straightforward to show that

$$\begin{array}{c} \underset{\mathbf{M} \in \mathbb{C}^{K \times K}}{\operatorname{argmin}} E\left[\|\mathbf{b} - \mathbf{Mr}\|^{2}\right] = \\ \underset{\mathbf{M} \in \mathbb{C}^{K \times K}}{\operatorname{argmin}} trace\left[\left(\mathbf{I} + \sigma^{-2} \boldsymbol{\Sigma} \mathbf{R} \boldsymbol{\Sigma}\right)\right]^{-1} \end{array}$$

Furthermore, if  $\Sigma$  is diagonal (usually the case for IID sequences), then

$$\mathbf{M} = \boldsymbol{\Sigma}^{-1} \left[ \mathbf{R} + \sigma^2 \boldsymbol{\Sigma}^{-2} \right]^{-1}$$
(5)

We usually interpret  $\sigma^2 \Sigma^{-2}$  as the inverse SNR metric per sublayer. The estimated variable  $\hat{\mathbf{b}}$  is formed when **M** is applied after a bank of Matched Filters **H**, i.e.,  $\hat{\mathbf{b}} = \mathbf{M}\mathbf{H}^H\mathbf{r}$ . This form of the MMSE estimator requires full channel state information and the sequel proposes an alternative solution.

#### B. MMSE Based Single User Detection: Explicit Training

Instead of using Matrix level estimation, we can take a vector approach by utilizing training sequences for each MIMO layer. The MMSE receiver for the  $k^{th}$  layer for the observed received spatial sequence  $\mathbf{r}[m]$  with m a discrete time index with coefficients  $\mathbf{w}_k = \frac{argmin}{\mathbf{w}_k} \left\{ E \left\{ \left| \mathbf{w}_k^H \mathbf{r} - b_k \right|^2 \right\} \right\}$ equate to the following filter  $\mathbf{w}_k = \mathbf{R}^{-1} E\{\mathbf{r}b_k\}$ . In the context of the linear model (3) the autocovariance matrix  $\mathbf{R} = E\{\mathbf{rr}^H\}$  is written as (6) assuming training symbols b and noise samples  $v_k$  are independent. This can be written as  $\mathbf{R} =$  $\mathbf{H}\Sigma\mathbf{H}^H + \Sigma_v$ , yielding the MMSE coefficients  $\mathbf{w}$  (6) where  $E\{\mathbf{r}b\} = \sigma_k^2\mathbf{H}\delta_k$  and  $\delta_k$  is the vector  $[0, 0, \dots, 1, 0, \dots, 0]^T$ where the position of the "1" denotes the layer of interest.

$$\mathbf{w}_{k} = \sigma_{k}^{2} \left( \mathbf{H} \boldsymbol{\Sigma} \mathbf{H}^{H} + \boldsymbol{\Sigma}_{v} \right)^{-1} \mathbf{H} \boldsymbol{\delta}_{k}$$
(6)

The Autocovariance Matrix  $\mathbf{R} = (\mathbf{H}\Sigma\mathbf{H}^H + \sigma_v^2\mathbf{I})$  and the Crosscorrelation vector is  $\mathbf{p}_k = \sigma_k^2\mathbf{H}\boldsymbol{\delta}_k$ , hence the error functional,  $J(\mathbf{w}_k)$  at the receiver terminal for the  $k^{th}$  layer is defined as

$$J(\mathbf{w}_k) = E\left\{ \left| b_k - \hat{b}_k(\mathbf{w}_k) \right|^2 \right\}$$
(7)

Expanding yields

$$J(\mathbf{w}_k) = E\left\{\theta_k^*(b_k - \hat{b}_k(\mathbf{w}_k))\right\} - E\left\{\hat{b}_k^*(\mathbf{w}_k)(b_k - \hat{b}_k(\mathbf{w}_k))\right\}$$

<u>Lemma 1</u>: For MMSE  $E\left\{\hat{b}_{k}^{*}(\mathbf{w}_{k})(b_{k}-\hat{b}_{k}(\mathbf{w}_{k}))\right\} = 0$ , the estimate  $\hat{b}_{k}(\mathbf{w}_{k})$  is orthogonal to the error (on average). *Proof*:  $E\left\{\left(\mathbf{w}_{k}^{H}\mathbf{r}\right)^{*}(b_{k}-\mathbf{r}^{H}\mathbf{w}_{k})\right\} = E\left\{\mathbf{w}_{k}^{H}\mathbf{r}b_{k}^{*}\right\} - E\left\{\mathbf{w}_{k}^{H}\mathbf{r}\mathbf{r}^{H}\mathbf{w}_{k}\right\}$ . Hence, with  $\mathbf{w}_{k}$  is constant, then

$$E\left\{\left(\mathbf{w}_{k}^{H}\mathbf{r}\right)^{*}\left(b_{k}-\mathbf{r}^{H}\mathbf{w}_{k}\right)\right\}=\mathbf{w}_{k}^{H}\mathbf{p}_{k}-\mathbf{w}_{k}^{H}\mathbf{R}\mathbf{w}_{k}$$

Corollary 1 : The MMSE solution is  $\mathbf{w}_k = \mathbf{R}^{-1}\mathbf{p}_k$ . Using the substitution  $\mathbf{w}_k^H \mathbf{p}_k - \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k = 0$  the MMSE  $E\left\{\hat{b}_k^*(\mathbf{w}_k)(b_k - \hat{b}_k(\mathbf{w}_k))\right\} = 0$ . When  $\mathbf{w}_k = \mathbf{R}^{-1}\mathbf{p}_k$ , it is easy to show that  $J(\mathbf{w}_k) = E\left\{b_k^*\left(b_k - \hat{b}_k(\mathbf{w}_k)\right)\right\}$ . Notinh  $J(\mathbf{w}_k) = \sigma_k^2 - \mathbf{w}_k^H \mathbf{p}_k$ , therefore,

$$J(\mathbf{w}_k) = \sigma_e^2[k] = \sigma_k^2 - \mathbf{p}_k^H \mathbf{R}^{-1} \mathbf{p}_k$$
(8)

where  $\sigma_e^2[k]$  is the variance of error for the  $k^{th}$  estimator.  $J(\mathbf{w}_k)$  is the error function, where if  $\mathbf{w}_k = \mathbf{R}^{-1}\mathbf{p}_k$  the minimum mean square error estimator is obtained.

Although the form in (8) differs from (5) it is easy to show that the same estimate  $\hat{b}[n]$  and thus MSE is achieved - i.e. that splitting the problem into a bank of individually optimized filters  $\mathbf{w}_k$  is the same as using the Matrix Transform in (5). Since the Matrix  $\sum$  is diagonal in (6), we can rewrite the weight vector as

$$\mathbf{w}_{k} = \mathbf{\Sigma}^{-1} \left( \mathbf{H} \mathbf{H}^{H} + \sigma_{v}^{2} \mathbf{\Sigma}^{-1} \right)^{-1} \mathbf{H} \boldsymbol{\delta}_{k}$$

with  $\Sigma = diag[\sigma_0^2, \sigma_1^2, \cdots, \sigma_{K-1}^2]$  with a 1 in the  $k^{th}$  position. If we define  $\mathbf{R} = \mathbf{H}\mathbf{H}^H$  and apply the Matrix Inversion Lemma, we obtain

$$\mathbf{w}_{k} = \frac{\sigma_{k}^{2}}{\sigma_{\nu}^{2}} \mathbf{h}_{k} - \sigma_{\nu}^{-2} \left[ \mathbf{H} \left( \mathbf{R} + \sigma_{v}^{2} \boldsymbol{\Sigma}^{-1} \right)^{-1} \mathbf{H}^{H} \right] \mathbf{h}_{k}$$

The scalar estimate  $\hat{b}_k = \mathbf{w}_k^H \mathbf{r}$  can be packed as a vector  $\hat{\mathbf{b}} = [\hat{b}_0, \hat{b}_1, \cdots, \hat{b}_{K-1}]^T$ , hence with

$$\hat{b}_{k} = \frac{\sigma_{k}^{2}}{\sigma_{\nu}^{2}} \mathbf{h}_{k}^{H} \mathbf{r} - \sigma_{\nu}^{-2} \mathbf{h}_{k}^{H} \mathbf{H} \left( \mathbf{R} + \sigma_{\nu}^{2} \boldsymbol{\Sigma}^{-1} \right)^{-1} \mathbf{H}^{H} \mathbf{r}$$

we can note the vector estimate  $\hat{\mathbf{b}}$  us given as

$$\hat{\mathbf{b}} = \sigma_{\nu}^{-2} \left[ \mathbf{\Sigma} \left( \mathbf{R} + \sigma_{v}^{2} \mathbf{\Sigma}^{-1} \right) - \mathbf{R} \right] \left( \mathbf{R} + \sigma_{v}^{2} \mathbf{\Sigma}^{-1} \right)^{-1} \mathbf{H}^{H} \mathbf{r}$$

If  $\Sigma = \mathbf{I}$ , then

$$\hat{\mathbf{b}} = \left(\mathbf{R} + \sigma_v^2 \mathbf{\Sigma}\right)^{-1} \mathbf{H}^H \mathbf{r}$$

which is the same as in (5). The regressor  $\mathbf{r}[n]$ , operated on by the filter **w** yields decision variable  $\hat{b}[n] = \mathbf{w}^H \mathbf{r}$  and steady state error given in (8), will be rewritten as

$$\sigma_e^2 = 1 - \boldsymbol{\delta}_k^T \mathbf{H}^H \left( \mathbf{H} \mathbf{H}^H + \gamma_{snr}^{-1} \mathbf{I} \right)^{-1} \mathbf{H} \boldsymbol{\delta}_k \tag{9}$$

assuming that symbol sequences are unit power  $\sigma_k^2 = 1$ , noise autocorrelation matrix  $\Sigma_{\nu} = \sigma_{\nu}^2 \mathbf{I}$ , and  $\gamma_{snr} = \frac{\sigma_{\mu}^2}{\sigma_{\nu}^2}$  the SNR for an AWGN equivalent channel. Using the Singular Value Decomposition on the matrix  $\mathbf{H} = \mathbf{A} \Theta \mathbf{B}^T$ , with  $\mathbf{A}$  the eigenvectors of  $\mathbf{H}\mathbf{H}^H$ ,  $\mathbf{B}$  the eigenvectors of  $\mathbf{H}^H\mathbf{H}$ , and  $\Theta$ the singular values with number of non-zero entries equal to  $rank(\mathbf{H})$ . Substituting into (9), noting that  $\mathbf{A}^{H}\mathbf{A} = \mathbf{I}$ ,  $\mathbf{B}^{H}\mathbf{B} = \mathbf{I}$ ,  $\mathbf{\Theta}\mathbf{\Theta}^{T} = \mathbf{\Lambda}$  and  $(\mathbf{A}\mathbf{\Theta}\mathbf{A}^{H})^{-1} = (\mathbf{A}\mathbf{\Theta}^{-1}\mathbf{A}^{H})$ , and  $\mathbf{\Theta}\mathbf{\Theta}^{T} = \mathbf{\Psi}$ , then

$$\sigma_e^2 = 1 - \boldsymbol{\delta}_k^T \mathbf{B} \left[ \boldsymbol{\Theta} \left( \boldsymbol{\Psi}^2 + \gamma_{snr}^{-1} \mathbf{I} \right)^{-1} \boldsymbol{\Theta} \right]^{-1} \mathbf{B}^H \boldsymbol{\delta}_k$$

Let  $\mathbf{\Lambda} = \mathbf{\Theta} \left( \mathbf{\Psi}^2 + \gamma_{snr}^{-1} \mathbf{I} \right)^{-1} \mathbf{\Theta}$  represent the diagonal matrix and  $\boldsymbol{\rho}_k = \mathbf{B}^T \boldsymbol{\delta}_k$ , then

$$\sigma_e^2 = 1 - \boldsymbol{\rho}_k^H \boldsymbol{\Lambda} \boldsymbol{\rho}_k$$

The Signal to Interference and Noise Ratio for the estimator is  $SINR = \frac{\rho_k^H \Lambda \rho_k}{\rho_k} = \frac{\rho_k^H \Lambda \rho_k}{\rho_k} = \frac{1 - \sigma_e^2}{\rho_e^2}$ 

$$SINR = \frac{\boldsymbol{\rho}_k^H \boldsymbol{\Lambda} \boldsymbol{\rho}_k}{\sigma_e^2} = \frac{\boldsymbol{\rho}_k^H \boldsymbol{\Lambda} \boldsymbol{\rho}_k}{1 - \boldsymbol{\rho}_k^H \boldsymbol{\Lambda} \boldsymbol{\rho}_k} = \frac{1 - \sigma_e}{\sigma_e^2}$$

The SINR per layer is related to the channel capacity for MMSE estimators.

<u>Lemma 2</u>: The capacity of a MIMO system with random signatures utilizing a bank of MMSE filters was derived in [15] and given by  $C = I_{max}(b_k; \hat{b}_k(\mathbf{w}_k)) = log\left(\frac{1}{\sigma_e^2[k]}\right)$ . Proof: If a vector of coefficients  $\mathbf{w}_k$  is chosen to minimize mean error, the error signal is orthogonal to the symbol estimate  $\left(E\left\{\hat{b}_k^*(\mathbf{w})(b_k - \hat{b}_k(\mathbf{w}))\right\} = 0\right)$ . When modulation symbols per layer,  $b_1, b_2, ..., b_k$  are Gaussian distributed random variables with zero mean, then

1)  $b_k(\mathbf{w}_k)$  is also Gaussian

2)  $\left(\hat{b}_k(\mathbf{w}_k) - b_k\right)$  is also zero mean Gaussian

<u>Lemma 3</u>: p(b) is a single variate distribution with standard deviation  $\sigma_b$ , the maximum entropy [14]  $H(b) = \sigma_b \log(\sqrt{2\pi e})$ .

<u>Lemma 4</u>: Given two independent variables  $\alpha$  and  $\beta$ . The entropy of  $\alpha$  given  $\beta$  is given by [16]  $H(\alpha|\beta) = H(\alpha)$ 

<u>Lemma 5</u>: For any two random variables  $\alpha$  and  $\beta$  [16],  $H(\alpha,\beta|\beta) = H(\alpha|\beta)$ 

The mutual information is given by:

$$I(b_k; \hat{b}_k(\mathbf{w})) = H(b_k) - H\left(b_k | \hat{b}_k(\mathbf{w})\right)$$

Hence,

$$I(b_k; \hat{b}_k(\mathbf{w})) = H(b_k) - H\left(b_k - \hat{b}_k(\mathbf{w})|\hat{b}_k(\mathbf{w})\right)$$
$$I_{max}(b_k; \hat{b}_k(\mathbf{w})) = \log(\sigma_b^2) - \log(E(n_e^2))$$

with the modulation alphabet normalized to unit rms  $\sigma_b = 1$ , then

$$I_{max}(b_k; \hat{b}_k(\mathbf{w}_k)) = C_k = \log\left(\frac{1}{\sigma_e^2[k]}\right)$$
(10)

where substituting (8) into (10) yields

$$C_k = \log\left(\frac{1}{1 - \mathbf{p}_k^H \mathbf{R}^{-1} \mathbf{p}_k}\right) \tag{11}$$

where  $0 \leq \mathbf{p}_k^H \mathbf{R}^{-1} \mathbf{p}_k < 1$ . In this paper we will be calculating the sum rate capacity for the spatially multiplexed system, i.e.

$$C = \sum_{k=0}^{K-1} \log\left(\frac{1}{1 - \mathbf{p}_k^H \mathbf{R}^{-1} \mathbf{p}_k}\right)$$
(12)

# C. AMUD Via Least Mean Squares

The Least Mean Square (LMS) algorithm [11] can be used instead. In it's standard form it is given as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \alpha e_k^* \mathbf{y} \tag{13}$$

where  $e_k^*$  is the conjugate of the instantaneous error and  $\alpha$  is the step size parameter which is [11] bounded by:

$$0 < \alpha < \frac{2}{\lambda_{max}}, and, \sum_{i=1}^{2M+1} \frac{\alpha \lambda_i}{2 - \alpha \lambda_i} < 1$$

where  $\lambda_i$  is the *i*<sup>th</sup> eigenvalue of the autocorrelation matrix **R**,  $\lambda_{max}$  is the maximum eigenvalue with U = 2M + 1 the number of estimator coefficients. The steady state MSE for the LMS algorithm is given by:

$$e(\infty) = \frac{J(\mathbf{w}_k)}{1 - \sum_{i=1}^{2M+1} \frac{\alpha \lambda_i}{2 - \alpha \lambda_i}}$$
(14)

Provided that  $\alpha$  is small, the steady state error is approximately (8)  $J(\mathbf{w}_k)$ . Typically a larger value of  $\alpha$  is used to speed up convergence until it reaches steady state and then a small value of  $\alpha$  is used to drive the steady state MSE closer to  $J(\mathbf{w}_k)$ .

#### **IV. SIMULATION PARAMETERS**

For simulations where the Bit Error Probability was estimated, we used 64 - QAM modulation. For the capacity analysis, unit power Gaussian sequences were utilized. The fading channel statistics were assumed to be Rayleigh distributed. For propagation purposes we assumed the distance between cooperative user terminals in transmitter to cooperative relay and cooperative relay to cooperative receiver cluster terminals were the same. For both Capacity and Bit Error Probability Analysis, power scaling was applied by the number of antenna terminals utilized in both directions, as well as scaling noise floor considering the Relay to Relay communication utilizing CDMA sequences. The Sum Rate Capacity for the single user (Spectral Efficiency) was left unscaled by the ratio of bandwidths (symbol rate divided by chipping rates) since the Relay to Relay channel is assumed multiple access. In each cluster, we assumed that the number of user terminals was 32 and that they were statistically uncorrelated fading channel wise. We modelled the transmitter and receiver clusters optimization wise when selecting the best 4 terminals out of 32. Our transmitter user terminal of interest used K = 4 antennas for spatial multiplexing. We assumed the receiver system was equipped with U = 8 antennas and assume that the same resources are employed by the relay terminals. We consider the application for our approach to be mm-wave channels within each cluster but SHF channels on the relaying side to account for obstacles and shadowing inevitable for longer haul mm wave communication with no line of sight. The local cluster could be designed for the 28GHz band while the relaying stage could employ an ISM band like 5.8GHz. The number of interferer's are treated as a simulation variable. The results were averaged over 1000 multipath channel realizations while Monte Carlo Simulation



Fig. 2. Receiver 1 Employed in this Paper



Fig. 3. Receiver 2 Employed in this Paper

was employed to estimate the Bit Error Probability (BEP). The estimator  $\mathbf{w}_k = [w_{k,0}, w_{k,1}, \cdots, w_{k,U-1}]^T \in \mathbb{C}^U$  are the coefficients for one DOF, producing a symbol estimate  $\hat{b}_k[m]$ =  $\mathbf{w}_k^H r[m]$  where k = 1, 2, ..., K the index for the Spatially Multiplexed Symbols and number of estimators. We design the estimators to recover the modulation symbols  $\hat{b}_k[m]$  but we choose to estimate symbol sampled or chip sampled sequences. The receiver algorithms are referred to as Receiver 1 Fig. (2) and Receiver 2 Fig. (3) respectively. The modelling parameters are given herewith for Receiver 1. Given  $\mathbf{H}_r \in \mathbb{C}^{U \times (K+I)}$ , let  $\Sigma_{H_n} = diag [\mathbf{H}_r(n)] \in \mathbb{C}^{(K+I) \times (K+I)}$  with  $\mathbf{H}_r(n)$  indicating the  $n^{th}$  row of the matrix  $\mathbf{H}_r$ . The bank of matched filters  $\mathbf{S}^T$ yields soft information  $\mathbf{R}\Sigma_{H_n}\mathbf{b} + \boldsymbol{\nu}_n$ . Letting  $\mathbf{A}_n = \mathbf{R}\Sigma_{H_n}$ with  $\mathbf{R} = \mathbf{S}^T \mathbf{S}$  a  $K \times (K + I)$  matrix and  $n = 1, 2, \dots, U$  an index referencing the **A** matrix on each receiver antenna, defin-ing  $\mathbf{B}_k = [\mathbf{A}_1(k)^T, \mathbf{A}_2(k)^T, \cdots, \mathbf{A}_U(k)^T]^T \in \mathbb{C}^{U \times (K+I)}$ with  $\mathbf{A}_u(k)$  the  $k^{th}$  row for the  $u^{th}$  antenna matrix  $\mathbf{A}_u$  enables a model to be build for each of the K layers transmitted / relayed. For the  $k^{th}$  layer, an adaptive multiuser detector can be utilized to estimate the transmitted symbols given  $\boldsymbol{\gamma}_k = \mathbf{B}_k \mathbf{b} + \boldsymbol{\nu}_k \in \mathbb{C}^{U \times 1}$  with the estimator for  $b_k$  utilizing the  $\gamma_k = \mathbf{b}_k \mathbf{b} + \boldsymbol{\nu}_k \in \mathbb{C}$  with the commute T is a second se into a tall matrix  $\mathbf{B}_k = \left[\mathbf{A}_1(k)^T, \mathbf{A}_2(k)^T, \cdots, \mathbf{A}_U(k)^T\right]^T \in$  $\mathbb{C}^{GU \times (K+I)}$  with the linear model banks of chips rather than symbols. For the  $k^{th}$  layer, the symbol estimate from  $\boldsymbol{\gamma}_k = \mathbf{B}_k \mathbf{b} + \boldsymbol{\nu}_k \in \mathbb{C}^{GU \times 1}$  is obtained with adaptive multiuser detection with the estimator for  $b_k$  utilizing the error metric  $b_k - \mathbf{w}_k^T \boldsymbol{\gamma}_k.$ 

# V. DISCUSSION

The Bit Error Probability results shown in Fig. (5) yield a comparison between the two different receiver approaches, and the information Capacity results are shown in Fig. (6). It was expected that under a no interference scenario the two receivers would yield near identical performance. The advantage with receiver 1 in low interference environments is that it could be simplified to just a bank of matched filters and a maximum ratio combiner if there was minimal multipath and orthogonal spreading sequence applied. The caveat to receiver 1 is that it can be quickly rendered underdetermined, in this example, with number of interferers > 5 unless one increases the receiver dimensionality(under the assumption of independent fading on each antenna to maximize the number of degrees of freedom). Altering the processing gain increases the performance saturation limit but will still yield link budgets as interference limited. The converse is true when examining receiver 2 which yields significantly higher interference tolerance due to both processing gain and receiver antenna dimensionality yielding greater degrees of freedom. The caveat to chip based optimization is the "SNR" will be quite low thus will be far more affected by phase noise and receiver noise margin / sensitivity issues - particularly regarding quantization. Without modelling or accounting for physical device limits, Fig. (4) indicates that the LMS algorithm utilized in this paper converges as well as it does in the case for receiver 1. The measure of information spectral efficiency was information bit rate normalized to single sided Nyquist Bandwidth. The difference between both receiver approaches, at least based on the simulation parameters presented in the paper, is approximately 10 bits/s/Hz (at a SNR of 30dB) subject to an interference constraint of 16 users. Bit Error Probability (BEP) wise, Receiver 2 achieves close to the interference free AWGN bound. Conversely, strong saturation in performance is evident for Receiver 1.

### VI. CONCLUSION

Two Receiver based optimization schemes are investigated applying Adaptive Single User Receiver Algorithms. They are demonstrated in this paper in context of Cooperative MIMO SHF broadcasting with applications envisaged in cases where the local MIMO channels are under a poor reception power constraint, i.e. such as being obstructed between receiver and base-station. The cooperative relaying is employed so that the terminal need not radically adjust the information rate due to loss of spatial degrees of freedom or fidelity. The Adaptive Receivers investigated do not require Channel State Information (CSI) to perform estimation and this is advantageous in communication environments where interference will practically be unavoidable. The same algorithm that drives interference rejection at the receiver is utilized in forming a Mean Square Error Metric for initializing the Cooperative Communication Channels, which is an advantage of adopting



Fig. 4. LMS MSE vs Iterations for Receiver 1 and Receiver 2.



Fig. 5. Information Spectral Efficiency for Receiver 1 and Receiver 2 for # Receiver antennas U = 8, # Relay Transmit antennas K = 4 per Relay, # Interferers I = 4, 8, 16, and spreading factor G = 32, 64.

an adaptive MMSE approach. The adaptive algorithm utilized in this paper is investigated for both chip-level and symbollevel optimization where it noted that when applying chip-level optimization, the system is more robust against interference utilizing processing gain (rather than receiver dimensionality) as a metric and variable to combat interference given the number of transmitter antennas used are fixed.



Fig. 6. BER Comparison for Receiver 1 and Receiver 2 for # Receiver antennas U = 8, # Relay Transmit antennas K = 4 per Relay, # Interference I = 4, 8, 16, and spreading factor G = 32, 64.

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